Prof. A. W. Date
Date. 23-3-10
Lec-6-7

$$\beta = \frac{2m}{m+1}$$

$$B_f = \frac{V_w}{U_\infty} Re_x^{1/2} = \text{Const.}$$

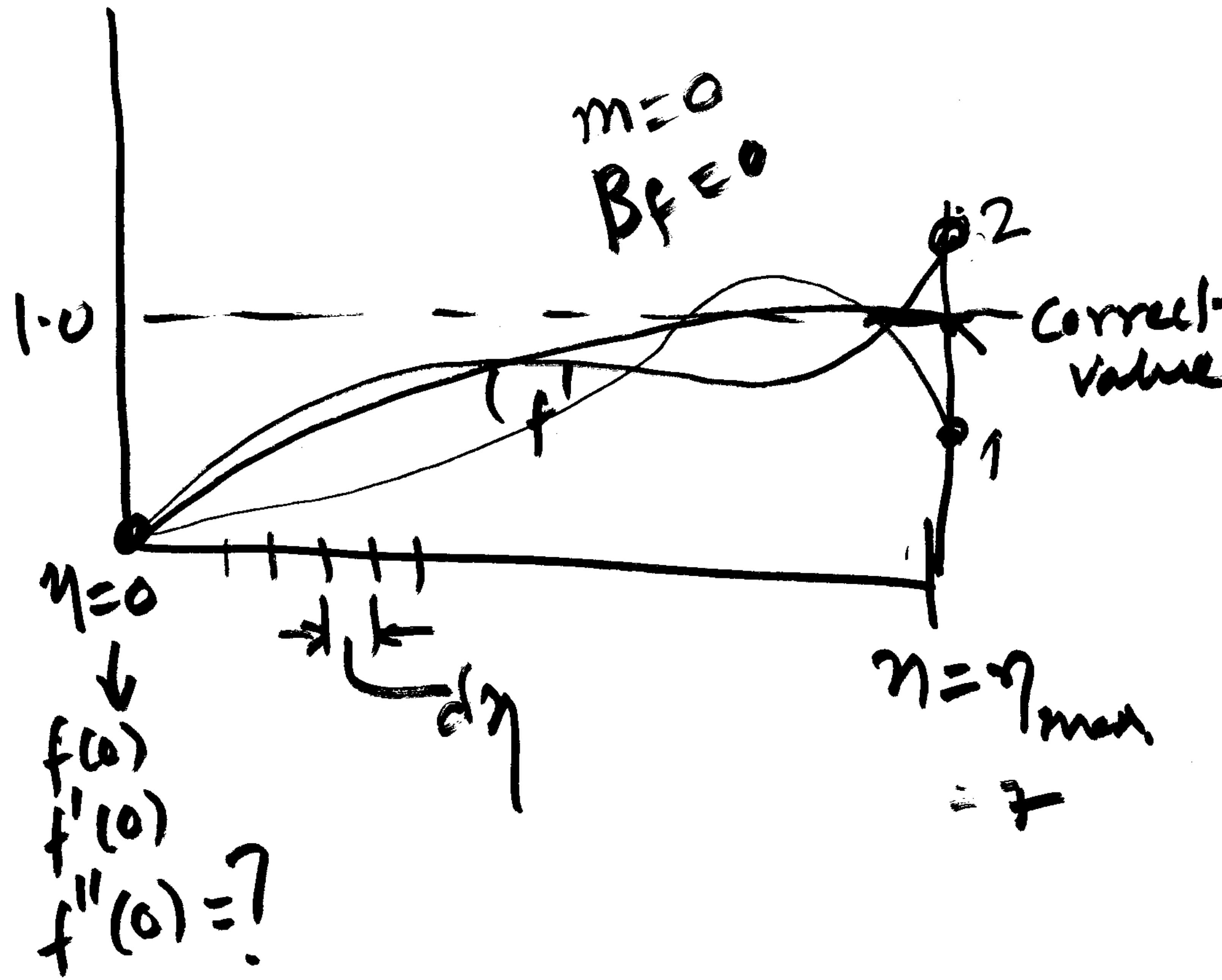
$$= \frac{V_w}{U_\infty}\sqrt{\frac{U_\infty x}{\nu}} = \text{Const.}$$

$$= V_w \sqrt{\frac{x}{\nu U_\infty}} = \text{Const.}$$

$$V_w \propto \sqrt{\frac{U_\infty}{x}} \propto x^{\frac{m-1}{2}}$$

$m=0$ $\quad V_w \propto \frac{1}{\sqrt{x}}$

$m=1$ $\quad V_w = \text{Const}$



m=0
βf=0
1·0
f'
2
1
correct value
η=0
dη
η=ηmax
=7
f(0)
f'(0)
f''(0)=?

$$\delta_1 = \int_0^{\delta} \left(1 - \frac{u}{u_\infty}\right) dy$$

$$\eta = y \cdot \sqrt{\frac{u_\infty}{\nu x}}$$

$$d\eta = dy \sqrt{\frac{u_\infty}{\nu x}}$$

$$= \int_0^{\eta_\delta} (1 - f') \cdot d\eta \cdot \sqrt{\frac{\nu x}{u_\infty}}$$

$$\eta_\delta = \delta \sqrt{\frac{u_\infty}{\nu x}}$$

$$\delta_1 \sqrt{\frac{u_\infty}{\nu x}} = \int_0^{\infty} (1 - f')\, d\eta_{\eta_{max}}$$

$$\approx \eta_{max}.$$

$$\delta_1^* = \frac{\delta_1}{x} \cdot \sqrt{\frac{u_\infty x}{\nu}} = \underline{\underline{\frac{\delta_1}{x} Re_x^{1/2}}} = \int_0^{\infty} (1 - f')\, d\eta$$

$$\delta^* = \frac{\delta}{x} \sqrt{\frac{u_\infty x}{\nu}} \rightarrow \eta_{max} \text{ at } f'(\eta) = .99$$

$$\delta_2^* = \frac{\delta_2}{x} \sqrt{\frac{u_\infty}{\nu x}} = \int_0^{\infty} f' (1 - f')\, d\eta$$

$$\delta_1 = \int_0^\infty \left(\frac{\rho_\infty u_\infty - \rho u}{\rho_\infty u_\infty} \right) dy \Bigg|^{\rho = \rho_\infty}$$

$$= \int_0^\infty \left(1 - \frac{u}{u_\infty} \right) dy$$

$$= \int_0^\delta \left(1 - \frac{u}{u_\infty} \right) dy + \cancel{\int_\delta^\infty \left(1 - \frac{u}{u_\infty} \right) dy} = 0$$

$$f''' + \frac{1}{2} f f'' = 0 \qquad m = 0$$

$$f'(0) = 0 \qquad f(0) = B_f \times \frac{2}{m+1} = 0$$

$$f'(\infty) = 1$$

$$f'(\eta) = u/u_\infty$$

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} \qquad \underline{\underline{u_\infty = c x^m}}$$

$$\underline{\eta} = y\left[\sqrt{\frac{c}{\nu}}\cdot x^{\frac{m-1}{2}}\right]$$

$$\frac{dP_\infty}{dx} = -\rho u_\infty \frac{du_\infty}{dx}$$

$$= -\rho c x^m \cdot c \cdot m \cdot x^{m-1}$$

$$= -\rho c^2 m \cdot \underline{\underline{x^{2m-1}}}$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$= \mu \cdot u_\infty \frac{\partial (u/u_\infty)}{\partial y}\Big|_{y=0}$$

$$= \mu u_\infty \cdot \frac{\partial f'}{\partial y}\Big|_{y=0}$$

$$= \mu \cdot u_\infty \frac{\partial f'}{\partial \eta}\Big|_{\eta=0} \frac{\partial \eta}{\partial y}$$

$$= \mu \cdot u_\infty \sqrt{\frac{u_\infty}{\nu x}} \cdot f''(0)$$

$$C_{fx} = \frac{\tau_w x}{\rho u_\infty^2/2} = \underline{2 f''(0) \cdot Re_x^{-0.5}}$$

$$\psi = f(\eta)\sqrt{\nu u_\infty x}$$

$$n(x) = \sqrt{\nu u_\infty x} = \sqrt{\nu c x^{m+1}}$$

$$= \sqrt{\nu c}\; x^{\frac{m+1}{2}}$$

$$\frac{dn}{dx} = \sqrt{\nu c}\cdot\left(\frac{m+1}{2}\right)\cdot x^{\frac{m-1}{2}}$$

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} = y\sqrt{\frac{c}{\nu}}\cdot x^{\frac{m-1}{2}}$$

$$\frac{d\eta}{dx} = y\sqrt{\frac{c}{\nu}}\cdot\frac{m-1}{2}\cdot x^{\frac{m-3}{2}}$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}\left(f(\eta)\cdot n\right)$$

$$= -\left[f\frac{dn}{dx} + n\frac{df}{d\eta}\cdot\frac{d\eta}{dx}\right]$$

$$= -\left[f\frac{dn}{dx} + n f'\frac{d\eta}{dx}\right]$$

$$\frac{v}{U_\infty}\sqrt{\frac{U_\infty x}{\nu}} = \frac{v}{U_\infty} Re_x^{1/2}$$

$$= -\left(\frac{m+1}{2}\right)\left[f + \frac{(m-1)}{(m+1)}\cdot f'(\eta)\right]$$

$$(2-\beta) f''' + \frac{f}{\cancel{\sqrt{2-\beta}}} \cdot \cancel{\sqrt{2-\beta}}\, f'' + \beta \cdot (1-f'^2) = 0$$

$$(2-\beta) f''' + f f'' + \beta (1-f'^2) = 0$$

$$\beta = \frac{2m}{m+1}$$

$$f''' + \left(\frac{m+1}{2}\right) f f'' + m(1-f'^2) = 0$$

$$z''' + z z'' + \beta(1 - z'^2) = 0$$

$$f'(\eta) = z'(\bar{\eta}) = \frac{u}{u_\infty}$$

$$z'' = \frac{dz'}{d\bar{\eta}} = \frac{d f'(\eta)}{d\bar{\eta}}$$

$$\eta = \bar{\eta}\sqrt{2-\beta}$$

$$= \frac{df'}{d\eta} \cdot \frac{d\eta}{d\bar{\eta}}$$

$$\begin{cases} z'' = \sqrt{2-\beta}\, f'' \\ z''' = (2-\beta)\, f''' \\ z = \dfrac{f}{\sqrt{2-\beta}} \end{cases}$$

$$U_\infty = c\,x^{\frac{\beta}{2-\beta}}$$

$$= c\,x^{m} \qquad m = \frac{\beta}{2-\beta}$$

$$\beta = \frac{2m}{m+1}$$

$$\bar{\eta} \rightarrow \eta = y\sqrt{\frac{U_\infty}{\nu x}} = \bar{\eta}\sqrt{2-\beta}$$

$$f'(\eta) \equiv Z'(\bar{\eta}) = \frac{u}{U_\infty}$$

$$2\beta_1 - \underline{\underline{\beta_2}} = \frac{2n}{\nu u_\infty}\frac{dn}{dx} - \underline{\underline{\frac{n^2}{\nu u_\infty^2}\frac{du_\infty}{dx}}}$$

$$= \frac{d}{dx}\left[\frac{n^2}{\nu u_\infty}\right]$$

$$\boxed{\frac{n^2}{\nu u_\infty} = (2\beta_1 - \beta_2)x}$$

$$\beta_2 = \frac{n^2}{\nu u_\infty^2}\frac{du_\infty}{dx} = (2\beta_1 - \beta_2)\frac{x}{u_\infty}\frac{du_\infty}{dx}.$$

$$\int\frac{du_\infty}{u_\infty} = \left(\frac{\beta_2}{2\beta_1 - \beta_2}\right)\int\frac{dx}{x}$$

$$\ln u_\infty = (\quad)\ln x$$

$$y=0 \quad \bar{\eta}=0 \qquad \underline{u=0=Z'(0)} \qquad \psi \; \underline{= \eta(x)\cdot Z(\bar{\eta})}$$

$$V_w = -\frac{\partial \psi}{\partial x}\Big|_{\bar{\eta}=0}$$

$$= -\left[\eta(x)\frac{dZ}{d\eta} + Z\cdot\frac{d\eta}{dx}\right]_{\bar{\eta}=0}$$

$$= -\left[\eta(x)\cancelto{0}{Z'(0)} + Z(0)\frac{d\eta}{dx}\right]$$

$$V_w = -Z(0)\frac{d\eta}{dx}$$

$$U_\infty \left(\frac{dz}{d\eta}\right)^2 \frac{du_\infty}{dx} - \frac{z}{\eta} u_\infty^2 \frac{d\eta}{dx} \cdot \frac{d^2 z}{d\eta^2}$$

$$= U_\infty \frac{du_\infty}{dx} + \left(\nu \frac{u_\infty^3}{\eta^2}\right) \frac{d^3 z}{d\eta^3}$$

$$u_\infty (z')^2 \frac{du_\infty}{dx} - \left(\frac{u_\infty^2}{\eta} \frac{d\eta}{dx}\right) z \cdot z''$$

$$= U_\infty \frac{du_\infty}{dx} + \left(\frac{\nu u_\infty^3}{\eta^2}\right) z'''$$

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = U_\infty \frac{dU_\infty}{dx} + \nu \frac{\partial^3 \psi}{\partial y^3}$$

$$\left[\frac{dF}{d\eta}\cdot\frac{dU_\infty}{dx} + U_\infty \cancel{\frac{d^2F}{d\eta^2}\frac{\partial \eta}{\partial x}}\right] U_\infty \frac{dF}{d\eta}$$

$$- \left[F\frac{d\eta}{dx} + \eta \cancel{\frac{dF}{d\eta}\frac{\partial \bar{\eta}}{\partial x}}\right]\frac{U_\infty^2}{\eta}\cdot\frac{d^2F}{d\eta^2}$$

$$= U_\infty \frac{dU_\infty}{dx} + \nu\cdot\frac{U_\infty^3}{\eta^2}\frac{d^3F}{d\eta^3}$$

$$\psi = n(x) Z(\bar{\eta})$$

$$\frac{\partial \psi}{\partial x} = Z \cdot \frac{dn}{dx} + n(x) \frac{dZ}{d\bar{\eta}} \cdot \frac{d\bar{\eta}}{dx}$$

$$\frac{u}{u_\infty} = \cancel{\frac{df}{d\bar{\eta}}} = \frac{1}{U_\infty}\frac{\partial \psi}{\partial y}$$

$$= \frac{1}{U_\infty} \cdot \frac{\partial \psi}{\partial \bar{\eta}} \cdot \frac{\partial \bar{\eta}}{\partial y}$$

$$1 = \frac{n}{U_\infty} \cdot \cancel{\frac{df}{d\bar{\eta}}} \cdot \frac{\partial \bar{\eta}}{\partial y}$$

$$\frac{\partial \bar{\eta}}{\partial y} = \frac{u_\infty(x)}{n(x)} = S(x)$$

$$\bar{\eta} = y \times \underline{s(x)}$$

$$\underline{\underline{U_0(x)}}$$

$$\frac{\partial \psi}{\partial y} = u \quad \frac{\partial \psi}{\partial x} = -v$$

$$\frac{\partial \psi}{\partial y} \cdot \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = u_0 \frac{d u_0}{dx} + \nu \frac{\partial^3 \psi}{\partial y^3}$$

$$\psi(x, y) = \underline{\underline{n(x)}} \cdot z(\bar{\eta})$$

$$\frac{\partial \psi}{\partial \bar{\eta}} = n \cdot \frac{dz}{d\bar{\eta}}$$

$$\frac{u}{U_0} = \frac{dz}{d\bar{\eta}} = z'$$